Math 155, ***Lecture Notes- Bonds*** **Name____________**

Section 9.7 *Taylor Polynomials and Approximations*

In this section, we will see how we can create special polynomial functions that can be used as approximations for other elementary functions. We will be using derivatives of an elementary function, we'll call this function $y = f(x)$, and we'll be expanding a polynomial function, we'll call this polynomial $y = P_n(x)$, about a fixed **center** point that we will call $(c, f(c))$. The original elementary function and this special polynomial will have behaviors that are extremely similar near the center point, $(c, f(c))$, and the more terms we use to expand our polynomial, the greater the accuracy we have for approximating $y = f(x)$, using $y = P_n(x)$.

Definitions of *n*th Taylor Polynomial and *n*th Maclaurin Polynomial

If f has n derivatives at c, then the polynomial

$$P_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \cdot \cdot \cdot + \frac{f^{(n)}(c)}{n!}(x-c)^n$$

is called the ***n*th Taylor polynomial for *f* at *c*.** If $c = 0$, then

$$P_n(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdot \cdot \cdot + \frac{f^{(n)}(0)}{n!}x^n$$

is also called the ***n*th Maclaurin polynomial for *f*.**

Ex. 1: Find the 6th degree Taylor Polynomial for $f(x) = \ln(1+x)$, centered at $c = 0$.

More Ex. 1:

Ex. 2: Find the 6^{th} degree Taylor Polynomial for $f(x) = \ln(1+x)$, centered at $c = 1$.

Ex. 3: Find the 4^{th} degree Maclaurin Polynomial for $f(x) = e^{3x}$.

Ex. 4: Use a 4^{th} degree Maclaurin Polynomial to approximate $\ln(1.05)$.

Is there a theorem about this?

THEOREM 9.19 Taylor's Theorem

If a function f is differentiable through order $n + 1$ in an interval I containing c, then, for each x in I, there exists z between x and c such that

$$f(x) = f(c) + f'(c)(x - c) + \frac{f''(c)}{2!}(x - c)^2 + \cdots + \frac{f^{(n)}(c)}{n!}(x - c)^n + R_n(x)$$

where

$$R_n(x) = \frac{f^{(n+1)}(z)}{(n + 1)!}(x - c)^{n+1}.$$

NOTE: $\left|R_n(x)\right| \le \frac{|x-c|^{n+1}}{(n+1)!} \bullet \max\left|f^{n+1}(z)\right|$

where $\max\left|f^{n+1}(z)\right|$ is the maximum value of $f^{n+1}(z)$ between x and c.

"When applying Taylor's Theorem, you should not expect to be able to find the exact value of z. (If you could do this, approximation would not be necessary.) Rather, you try to find bounds for $f^{n+1}(z)$ from which you are able to tell how large the remainder $R_n(x)$ is."

Vocabulary: $f(x) = P_n(x) + R_n(x)$

$$Error = \left|R_n(x)\right| = \left|f(x) - P_n(x)\right|$$

Ex. 5: How good was our $P_4(x)$ Maclaurin approximation of $\ln(1.05)$?

Ex. 6: Estimate $e^{0.3}$ with an error less than 0.001. Determine the degree if this Maclaurin Polynomial.

More Ex. 6:

Ex. 7: Determine the degree of the Maclaurin Polynomial required for the error in the approximation of $\cos(0.1)$ to be less than 0.001.

More Ex. 7: